Parameter mismatches and perfect anticipating synchronization in bidirectionally coupled external cavity laser diodes

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We study *perfect* chaos synchronization between two bidirectionally coupled external cavity semiconductor lasers and demonstrate that mismatches in laser photon decay rates can explain the experimentally observed anticipating time in synchronization.

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Chaos synchronization [1] is of fundamental importance in a variety of complex physical, chemical and biological systems [2]. Application of chaos synchronization has been advanced in secure communications, optimization of nonlinear system performance, modeling brain activity and pattern recognition [2]. Time-delay systems are ubiquitous in nature, technology and society because of finite signal transmission times, switching speeds, and memory effects [3]. Therefore the study of chaos synchronization in these systems is of considerable practical significance. Because of their ability to generate high-dimensional chaos, time-delay systems are good candidates for secure communications based on chaos synchronization. In this context particular emphasis is given to the use of chaotic external cavity semiconductor lasers [4].

Following the discovery of anticipating synchronization by Voss [5], Masoller studied theoretically and numerically anticipating synchronization in unidirectionally coupled lasers and showed that the anticipating time should be equal to the difference between round-trip time of the light in the transmitter's external cavity and the time of flight between the lasers [6]. The first experimental observation of anticipating synchronization between two bidirectionally coupled external cavity laser diodes was reported recently [7]. In a bidirectional system such anticipating chaos is rather robust but it has proved to be rather difficult to obtain a reproducible demonstration of anticipating synchronization in unidirectionally coupled laser diodes. It is noted also that it was found experimentally [7] that the solitary receiver laser anticipates the chaotic transmitter by the time of flight between the lasers. At present there is no full theoretical explanation of the experimental results.

In this Brief Report we demonstrate the possibility of *perfect* chaos synchronization between two bidirectionally coupled external cavity semiconductor lasers. We demonstrate that mismatches in laser photon decay rates can explain the time of flight anticipating time synchronization between the laser diodes. In a recent paper [8] we have demonstrated the role which parameter mismatches play in the explanation of the coupling-delay lag time synchronization in unidirectionally coupled systems. Knowledge of the exact time shift between the synchronized states is of obvious considerable practical importance for the recovery of message at the receiver of a chaotic communication system [4,9].

An appropriate framework for treating the evolution of the electric field of external cavity laser diodes is provided by the widely utilized Lang-Kobayashi equations [10]. Suppose that the master laser is described by the equations

$$\begin{aligned}\frac{dE_1}{dt} &= \frac{(1+\imath\alpha_1)}{2}\left(\frac{G_1(N_1-N_{01})}{1+s_1|E_1|^2}-\gamma_1\right)E_1(t)\\ &\quad +k_1E_1(t-\tau_1)\exp(-\imath\omega\tau_1)\\ &\quad +k_3E_2(t-\tau_2)\exp(-\imath\omega\tau_2),\\ \frac{dN_1}{dt} &= J_1-\gamma_{e1}N_1-\frac{G_1(N_1-N_{01})}{1+s_1|E_1|^2}|E_1|^2, \end{aligned}\tag{1}$$

is coupled bidirectionally with the slave laser described by equations

$$\begin{aligned}\frac{dE_2}{dt} &= \frac{(1+\imath\alpha_2)}{2}\left(\frac{G_2(N_2-N_{02})}{1+s_2|E_2|^2}-\gamma_2\right)E_2(t)\\ &\quad +k_2E_2(t-\tau_1)\exp(-\imath\omega\tau_1)\\ &\quad +k_3E_1(t-\tau_2)\exp(-\imath\omega\tau_2),\\ \frac{dN_2}{dt} &= J_2-\gamma_{e2}N_2-\frac{G_2(N_2-N_{02})}{1+s_2|E_2|^2}|E_2|^2, \end{aligned}\tag{2}$$

where $E_{1,2}$ are the slowly varying complex fields for the master and slave lasers, respectively; $N_{1,2}$ are the carrier densities; $\gamma_{1,2}$ are the cavity losses; $\alpha_{1,2}$ are the linewidth enhancement factors; $G_{1,2}$ are the optical gains; $k_{1,2}$ are the feedback levels; k_3 is the coupling rate; ω is the optical frequency without feedback (no frequency detuning between the two lasers); τ_1 is the round-trip time in the external cavity; τ_2 is the time of flight between the master laser and the slave laser-coupling delay time; $J_{1,2}$ are the injection currents; $\gamma_{e1,e2}^{-1}$ are the carrier lifetimes; $s_{1,2}$ are the gain saturation coefficients.

We show that mismatches between the master and slave laser photon decay rates $\gamma_1 \neq \gamma_2$ can result in the experimentally observed time of flight anticipating synchronization

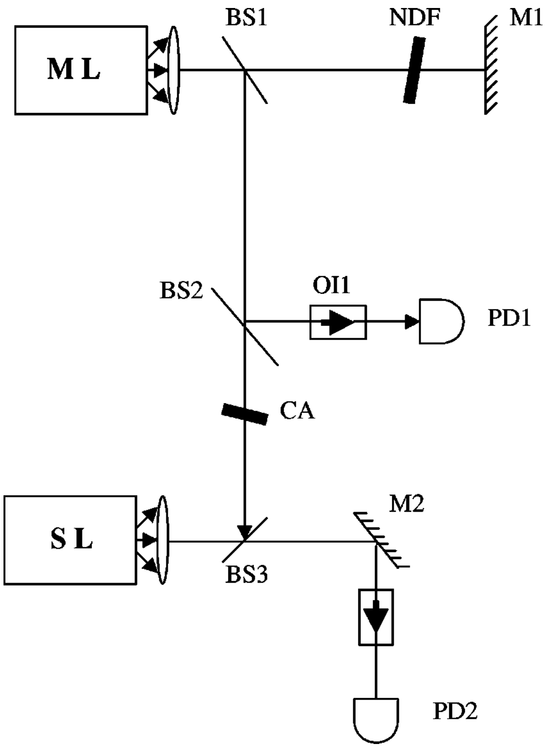

FIG. 1. ML, master laser; SL, slave laser; BS1-3, beam splitters; NDF, neutral density filter; OI1-2, optical isolators; M1-2, mirrors; CA, coupling attenuator; PD1-2, photodetectors.

time. Mathematically the intensities of the master and slave lasers should be related by

$$I_1 = I_{2,\tau_2}. \tag{3}$$

Throughout this paper $x_\tau \equiv x(t-\tau)$. We also assume an analogous synchronization manifold for the carrier densities: $N_1 = N_{2,\tau_2}$. Using Eq. (2) we write the dynamical equation for the E_{2,τ_2} in the following manner:

$$\frac{dE_{2,\tau_2}}{dt} = \frac{(1+\imath\alpha_2)}{2}\left(\frac{G_2(N_{2,\tau_2}-N_{02})}{1+s_2|E_{2,\tau_2}|^2} - \gamma_2\right)E_{2,\tau_2} + k_3 E_{1,2\tau_2}\exp(-\imath\omega\tau_2).$$

In accordance with experiments [7] we consider the case of a solitary slave laser, i.e., $k_2 = 0$. The experimental setup from Ref. [7] is reproduced in Fig. 1. It is assumed that, except for the photon decay rates, the laser parameters are identical. Then we find that the equations for E_1 and E_{2,τ_2} will be identical and therefore perfect synchronization (3) will be possible if conditions

$$\frac{(1+\imath\alpha)}{2}\gamma_1 - k_3\exp(-\imath\omega\tau_2) = \frac{(1+\imath\alpha)}{2}\gamma_2, \tag{4}$$

(where $\alpha = \alpha_1 = \alpha_2$) and

$$k_1 E_1(t-\tau_1)\exp(-\imath\omega\tau_1) = k_3 E_1(t-2\tau_2)\exp(-\imath\omega\tau_2) \tag{5}$$

are met. One can easily rewrite condition (4) in the more appealing form

$$(\gamma_1 - \gamma_2)^2 = \frac{4k_3^2}{1+\alpha^2}. \tag{6}$$

In general, it is unreasonable to impose conditions on the chaotic transmitter itself as in Eq. (5). Fortunately, at least in certain cases no such restrictions are needed. For example, assuming $\tau_1 = 2\tau_2$ one finds that the perfect synchronization manifold Eq. (3) exists for $k_1 = k_3$ and $\omega\tau_2 = 2\pi n$ (where $n = 0,1,2,\ldots$). We also notice that the synchronization manifolds $I_1 = I_{2,\tau_2}$ and $I_1 = I_{2,\tau_1-\tau_2}$ are identical for $\tau_1 = 2\tau_2$, but in general, $I_1 = I_{2,\tau_1-\tau_2}$ is not the synchronization manifold. Our approach, in principle allows also for anticipating synchronization with time of flight anticipation time, even in the case when $\tau_1 = \tau_2$, as was reported experimentally [7]. The thrust of this communication is to demonstrate the possibility of achieving perfect chaos synchronization in the physical system studied in Ref. [7]. It is clearly of some importance to examine the stability of the identified synchronized state. However that would require substantial numerical investigations which are beyond the scope of the present Brief Report. We intend to present results of detailed numerical simulations of Eqs. (1) and (2) in a separate work.

We conclude this Brief Report with the following remarks. Usually parameter mismatches are considered to have a detrimental effect on the synchronization quality between coupled identical systems: in the case of small parameter mismatches the synchronization error does not decay to zero with time, but can show small fluctuations about zero or even a nonzero mean value [6]. Larger values of parameter mismatches can even result in the loss of synchronization. However it appears that in reality the relation between chaos synchronization in time-delayed systems and parameter mismatches is quite intricate and complex. In a recent paper [8] we have shown that parameter mismatches can change the time shift between the synchronized states; moreover we have presented an example where the presence of parameter mismatches is the *only way to achieve chaos synchronization* between two unidirectionally coupled time-delayed systems. In the present Brief Report we have shown that perfect anticipating synchronization between two bidirectionally coupled external cavity laser diodes is possible in the presence of parameter mismatches. As knowledge of the time shift between the synchronized states is of considerable practical importance for the message recovery in communications and information processing using chaos control methods, further research on relation between chaos synchronization and parameter mismatches would be desirable.

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